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Bundling with Customer Self-Selection: A Simple Approach to Bundling Low-Marginal-Cost Goods

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With declining costs of distributing digital products comes renewed interest in strategies for pricing goods with low marginal costs. In this paper, we evaluate *customized bundling*, a pricing strategy that gives consumers the right to choose up to a quantity M of goods drawn from a larger pool of N different goods for a fixed price. We show that the complex mixed-bundle problem can be reduced to the customized-bundle problem under some commonly used assumptions. We also show that, for a monopoly seller of low marginal cost goods, this strategy outperforms individual selling ($M = 1$) and pure bundling ($M = N$) when goods have a positive marginal cost or when customers have heterogeneous preferences over goods. Comparative statics results also show that the optimal bundle size for customized bundling decreases in both heterogeneity of consumer preferences over different goods and marginal costs of production. We further explore how the customized-bundle solution is affected by factors such as the nature of distribution functions in which valuations are drawn, the correlations of values across goods, and the complementarity or substitutability among products. Altogether, our results suggest that customized bundling has a number of advantages—both in theory and practice—over other bundling strategies in many relevant settings.

Key words: information goods; digital goods; pricing; bundling; self-selection; Internet
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1. Introduction

The emergence of the Internet as a low-cost, mass-distribution medium has renewed interest in pricing structures for information and other digital goods (Choi et al. 1997, Shapiro and Varian 1998). Of recent interest are settings in which firms are attempting to sell a large number of low-marginal-cost goods to consumers who have different preferences over the value of the individual goods. This type of setting naturally arises in digital product settings, such as cable television, digital music, modular software, and news or journal articles, wherein different consumers desire different products within a broader class but the cost of distribution is similar across goods and small relative to value. Although it is now possible to efficiently sell individual goods separately for even small payments (Metcalfe 1996), firms may be able to generate greater profits by engaging in bundling, wherein large numbers of goods are sold as a unit.

In theory, for N goods, firms could offer up to $(2^N - 1)$ possible bundles, each at a (possibly) different price. However, this bundle composition problem is known to be computationally intractable and difficult to solve in closed form except for small numbers of goods (Hanson and Martin 1990). Moreover, this exhaustive bundling strategy potentially imposes significant burdens on customers to evaluate a large menu of bundles and requires that firms have exact reservation prices for all possible bundles and all consumers. Consequently, offering an exhaustive set of bundle compositions is rarely implemented in practice. Recent work (Bakos and Brynjolfsson 1999, referred to hereafter as BB) has shown that when the marginal cost of each good is low and valuation is determined by a common distribution function across goods, offering all goods for a fixed price ("pure bundling") is optimal. This greatly simplifies the bundling and pricing problem. However, less is known about situations in which it may be optimal to bundle large numbers of goods but marginal costs and consumer preferences are such that pure bundling is inefficient. These types of situations arise when goods have a small but nonnegligible marginal cost[1] and when consumers value only a subset of all available goods.

In this paper, we analyze a pricing approach that generalizes existing results on information goods pricing while preserving simplicity and analytical tractability, which we term *customized bundling*. A customized bundle is a consumer's right to buy a choice of up to M goods from among a larger set N for a fixed price p. This mechanism has been proposed as a price discrimination mechanism (Bakos and Brynjolfsson 2000, Shapiro and Varian 1998) and has been previously analyzed on a small scale using mixed-integer programming (Chen 1998). More recently, researchers have studied the synonymous concept of "generalized subscriptions" for academic journal articles using field experiments and numerical analysis (MacKie-Mason and Riveros 2000, MacKie-Mason et al. 2000, Riveros 2000). However, there is very limited analytical work on this topic and much of the existing research is specific to a particular context. Our objective in this work is to provide a more general analytical framework and analysis of customized bundling that allows for a broader conception of customer heterogeneity and interrelationships among values of different goods such as complementarity and correlation. In the past, research on these issues has focused principally on the pure bundling context or settings with a small set of goods.

In the last decade, interest in customized bundling strategies has significantly increased, especially as applied to the sale of information goods. Firms such as the *New York Times* (article archives), Pressplay (digital music), Netflix (DVDs), and Verizon (optional telecommunications services) have all been experimenting a customized bundling pricing scheme for distribution of digital content. However, other companies selling similar products such as academic journal publishers (article archives), WSJ.com (article archives), and iTunes (music downloads) utilize a fixed price per unit (individual sale) strategy. Others, such as cable and satellite television providers, create predesignated "packages" that bundle a fixed collection of services and thereby restrict consumer choice. Given that the range of products amenable to bundled sale is increasing significantly over time, especially with the increasing trend to price information services offered over the Internet (e.g., reviews, price search, news), the customized bundling strategy is increasingly viable as a pricing and distribution approach. The goal of this paper is to explore the conditions under which the customized bundling strategy is advantageous and examine how market conditions affect the optimal size and price of customized bundles.

Our analysis first considers the general relationship between mixed bundling and customized bundling, and then characterizes the customized bundling solution under different assumptions about cost, value, and consumer preferences. We also compare customized bundling to the "traditional" problem alternatives such as unit sale, two-part tariffs, and pure bundling. Our results suggest that if consumer demand can be characterized in ways consistent with the assumptions used in some previous work on information goods pricing, then the mixed-bundle problem can be reduced to a simple problem of nonlinear pricing. This allows the application of known results to solve otherwise very complicated and general bundling problems.[2] In addition, because customized bundling contains unit sale and pure bundling as extreme cases, we can compare the customized bundling approach to these more common pricing mechanisms. Collectively, these results can potentially delineate the benefits and limitations of customized bundling as a pricing approach and lead to greater use of these and related pricing methods in practice or for further theory development.

2. Previous Literature

The literature on bundling has a long history beginning with the observation by Stigler (1963) that bundling can increase sellers' profits when consumers' reservation prices for two goods are negatively correlated. In the two-good case, offering both a two-good bundle as well as the individual items ("mixed bundling") is typically optimal (Adams and Yellen 1976, McAfee et al. 1989). This is because bundling reduces heterogeneity in consumer valuations, enabling a monopolist to price discriminate better (Schmalensee 1984, Salinger 1995), while still capturing residual demand through unit sale. These insights extend beyond this case to situations when goods can be complements or substitutes (Venkatesh and Kamakura 2003).

Other work has extended the bundling literature to consider multiple goods as well as multiple consumer types. Spence (1980) generalized the principles of the

[1] These costs need not be limited to production or reproduction cost. For example, they could include distribution costs, a cost of monitoring or otherwise enforcing a price scheme (e.g., billing), and consumer effort in selecting products. These costs can be significant, even for goods with low or zero marginal production cost.

[2] Customized bundling may provide a reasonable approximation to the general bundling problem, even where the assumptions required for customized bundling to be optimal are violated. For instance, under the assumptions in Hanson and Martin (1990, Table 2, p. 164), customized bundling approximates the exact solution within 2% of its profit. However, the approximation is less accurate when marginal costs vary considerably across goods and across bundles.

single-product pricing problem to the case of several products using a nonlinear programming formulation and showed some cases in which the problem can be solved in closed form. Other tractable analytical solutions have been found for a variety of special cases such as linear utility (McAfee and McMillan 1988) or when valuations across different consumers can be ordered in specific ways or satisfy certain separability conditions (Armstrong 1996, Sibley and Srinagesh 1997, Armstrong and Rochet 1999). These papers have found additional general results. For instance, Armstrong (1996) found that it is usually optimal to leave some consumers unserved in order to extract more revenue from the other, higher-value consumers. Rochet and Chone (1998) found that it is sometimes optimal to induce a degree of "bunching," so that consumers with different tastes are forced to choose the same bundle of products. These papers provide a general structure for solving complicated bundling problems in closed form, although the complexity increases dramatically as more goods are considered, making it difficult to extend the methods to bundling problems with large numbers of items.

An alternative approach is to solve the problem directly using optimization or numerical methods. Hanson and Martin (1990) use mixed integer programming to determine optimal prices as well as the composition of product bundles targeted to different market segments. However, the complexity of the problem in their model grows exponentially as the number of goods increases, and their approach assumes a well-informed monopolist who knows consumers' reservation prices with certainty. Chung and Rao (2003) focus on the pure bundling situation, developing a product attribute model of consumer utility in bundling settings and applying it to find market segments and optimal bundle pricing. Other research has examined the optimality of bundling strategies for goods where values may be related, either through correlation in reservation prices or as complements or substitutes (Jedidi et al. 2003).

The approach we utilize is to identify conditions under which the bundling problem can be simplified. Bakos and Brynjolfsson (1999) show that pure bundling can often be optimal when marginal costs are sufficiently low, given some relatively weak conditions on preferences (identically distributed valuations). However, when pure bundles are not optimal, such as when consumers are budget constrained, when consumers do not value all goods or when marginal costs are significant, pure bundling can create substantial deadweight loss.[3]

There have been several studies that have considered large-numbers bundling problems in specific contexts related to information goods pricing. These studies generally find that engaging in a form of mixed bundling, in which a certain large bundle is offered alongside individual sale, dominates either strategy alone (Chuang and Sirbu 1999, Fishburn et al. 2000). In addition, these studies introduce the idea that allowing customers to self-select the goods in the bundle (rather than having the goods predesignated) can often improve outcomes while maintaining simplicity in the pricing mechanism (Chen 1998, Chuang and Sirbu 1999, MacKie-Mason and Riveros 2000). However, much of the insights of these works are based on experiments or numerical explorations (MacKie-Mason et al. 2000, Riveros 2000). Our contribution to this literature is to formally model this approach and to characterize in detail the behavior of the customized-bundle problem in the "interior" where neither individual sale nor pure bundling may be optimal. A by-product of this approach is that we contribute a different method for examining these more traditional pricing approaches.

3. Model

3.1. Introduction

The general setting we consider is a monopolist selling N goods. We are interested in examining the profitability of customized bundles for a monopolist, in which a consumer is allowed to choose up to M goods ($M \leq N$) for a single price p. In general, a monopolist may want to offer more than one customized bundle when facing heterogeneous customers. For notational simplicity, we will use $m \in [0, 1/N, 2/N, \ldots, 1]$ to represent a fraction of the total number of goods available and let $p(m)$ represent the price for a bundle of size m. In addition, for a function $p(m)$ we define the notation $f'(m)$ as $f(m) - f(m - 1/N)$ for $m \geq 1/N$, to be consistent with the discrete nature of m.

3.2. Multiproduct Nonlinear Pricing for Heterogeneous Consumers

We begin by defining a structure for the standard bundling problem in which customers demand at most one unit of each good. Consumers purchase

[3] The insight behind this shortcoming is straightforward. Suppose that there are a large number of consumers who have valuation for each of 10 goods drawn from the same distribution function. It is clear that if we offer a pure bundle, all consumers will obtain their most preferred goods, although not all will agree which ones they are. However, if we are constrained such that we can only sell, say, a five-good bundle, there are now 252 possible bundles that a consumer might want if they can only have five goods. If any single bundle among the 252 possible bundles is offered and valuations are uniformly distributed, on average only 1/252 of the customers will receive their highest-valued goods, creating substantial deadweight loss. Only by offering every possible combination that consumers' desire would this deadweight loss disappear.

a bundle of goods $\mathbf{x} = \langle x_1, \ldots, x_j, \ldots, x_N \rangle$ (where the elements of $\mathbf{x}$ are binary variables, $x_j \in \{0, 1\}$, $j = 1, \ldots, N$ indicating the consumption for each component) over all N goods available. Consumers derive benefits from these goods, which leads to a willingness to pay (WTP) of $W(\mathbf{x})$, a weakly increasing function in all components of $\mathbf{x}$ with $W(\mathbf{0}) = 0$. Theoretically, there can be as many sets of consumer preferences as there are consumers. Let there be I distinct types of consumers indexed $i \in [1, 2, \ldots, I]$, each with a unique WTP function $W^i(\mathbf{x})$. The proportion of each consumer type in the population is denoted by α^i (where $\sum_{i=1}^{I} \alpha^i = 1$). If the price of a set of goods is $p(\mathbf{x})$, we could write the utility that a consumer i obtains from purchasing this bundle as $U^i(\mathbf{x}, p(\mathbf{x})) = W^i(\mathbf{x}) - p(\mathbf{x})$.[4] We denote the cost of providing a vector of goods $\mathbf{x}$ as $C(\mathbf{x})$, which is weakly increasing in all components of $\mathbf{x}$. Using this notation, the general bundling problem the monopolist faces is the determination of the set of bundles offered $\{\mathbf{x}\}$ and a set of prices $p(\mathbf{x})$ solving the well-known mixed-bundle pricing problem with heterogeneous consumers (Spence 1980):

$$\begin{aligned} &\max \sum_{i=1}^{I} \alpha^i [p(\mathbf{x}^i) - C(\mathbf{x}^i)] \\ &\text{s.t. IR: } W^i(\mathbf{x}^i) - p(\mathbf{x}^i) \geq 0 \quad \forall i, \\ &\quad\ \ \text{IC: } W^i(\mathbf{x}^i) - p(\mathbf{x}^i) \geq W^i(\mathbf{x}^j) - p(\mathbf{x}^j) \quad \forall i, j \neq i. \end{aligned} \tag{1}$$

The first set of constraints, individual rationality (IR), guarantees that if a consumer chooses to purchase a bundle, it provides nonnegative surplus (purchase is voluntary). The second set of constraints, incentive compatibility (IC), guarantees that a consumer segment receives at least as much surplus for purchasing the bundle intended for them as they would from choosing another bundle. Implicit in this assumption is that the monopolist cannot price discriminate by group; that is, it must be in the consumer's self-interest to purchase their intended bundle. This formulation treats the problem as a direct revelation mechanism where consumers reveal their "type" through their choice of product, which will yield the profit-maximizing solution for the monopolist (Myerson 1979). Note from this formulation that for I consumer groups and N products, the monopolist must determine the optimal set of I bundle compositions and prices out of $2^N - 1$ possibilities.

3.3. Customized Bundling

Our initial interest is in determining the conditions under which the complex bundle composition problem can be reduced to the much simpler customized-bundle problem—reducing the problem space from $(2^N - 1)$ to N possible bundles. Following the literature on information goods pricing, we will assume that the cost structure of providing goods to consumers depends only on the number and not on which goods are provided, thus $C(\mathbf{x}) = C(m)$, where $m = (1/N)\mathbf{x} \bullet \mathbf{1}$ ($\bullet$ denotes a vector dot product, and $\mathbf{1}$ is a vector of all 1s). We further assume that C is weakly increasing, with decreasing differences in m (that is, $C'(m) \geq 0$ and $C''(m) \leq 0$). In addition, $C(0) = 0$, consistent with the notion that the monopolist has already sunk any fixed cost necessary to produce these goods.

Before establishing these results, it is useful to introduce some additional notation. Let $w^i(m)$ represent the most a consumer of type i is willing to pay for mN goods (formally, $w^i(m) = \max_{\mathbf{x}} W^i(\mathbf{x})$ s.t. $\sum_{k=1}^{N} x_k \leq mN$). This implies that $w^i(0) = 0$, $w^{i\prime}(m) \geq 0$, and $w^{i\prime\prime}(m) \leq 0$ $\forall i$. Although there can exist as many as I such functions,[5] in general there can be less than I because different preferences $W(\mathbf{x})$ can yield the same expression for $w(m)$.[6] We can now formulate the customized-bundle problem as

$$\begin{aligned} &\max \sum_{i=1}^{I} \alpha^i [p(m^i) - C(m^i)] \\ &\text{s.t. IR: } w^i(m^i) - p(m^i) \geq 0 \quad \forall i, \\ &\quad\ \ \text{IC: } w^i(m^i) - p(m^i) \geq w^i(m^j) - p(m^j) \quad \forall i, j \neq i. \end{aligned} \tag{2}$$

This problem is the well-known nonlinear pricing problem with heterogeneous consumers (also known as second-degree price discrimination; see Tirole 1988, pp. 148–154). In addition to the mathematical formulation being identical, customized bundling is also intuitively similar to second-degree price discrimination because it accomplishes discrimination among different groups through customer self-selection from a menu of offerings.[7] This problem is much simpler than the general bundling problem (1) because it only requires a selection of a maximum of I prices from a total space of N possible customized bundles. In Result 1, we show the conditions required to make customized-bundle problem (2) yield the same profit as the general bundling problem (1):

Result 1. The customized-bundle solution $p(m)$ $\forall m$ yields the same profit and consumer choices as optimal mixed-bundle price schedule $\{\mathbf{x}^i, p(\mathbf{x}^i)\}$ $\forall i$,

[4] The assumptions on W guarantee that U obeys the normal properties of utility functions.

[5] In the context of information goods, it is reasonable to assume that I (number of consumer types) is much smaller than N (number of information goods offered).

[6] This is because we are mapping from a larger domain to a smaller domain.

[7] The key distinction is that the nonlinear pricing problem generally refers to different quantities of an identical good, while customized bundling refers to heterogeneous goods with similar valuations.

if for any optimal bundle $\mathbf{x}^i$ offered, where $\mathbf{x}^i \cdot \mathbf{1} = m^i N$, one of the following conditions hold:[8] (A) $w^j(m^i) \le w^i(m^i) = W^i(\mathbf{x}^i)\ \forall j$ or (B) $W^i(\mathbf{x}^j) = w^i(m^j)\ \forall j$.

The conditions in Result 1 rule out a mixed-bundle solution with different prices for the same number of goods. In addition, they assure that a customer who is free to choose any bundle of a given size, chooses the same bundle as they would under the optimal mixed-bundle problem and does not switch to another bundle of the same size that was not offered in the mixed-bundle solution.

A simple example meeting the conditions of Result 1(A) is when heterogeneous preferences over goods map to a single WTP in customized bundles, that is, $w^i(m) = w(m)\ \forall i,\ \forall m$.[9] Consider a setting where there are three goods (a, b, c) with a marginal cost per good of 1/4 and three consumers $(1, 2, 3)$ whose valuations are

	a	b	c	$w^i(1/3)$	$w^i(2/3)$	$w^i(3/3)$
1	0.1	0.4	1.0	1.0	1.4	1.5
2	0.4	1.0	0.1	1.0	1.4	1.5
3	1.0	0.1	0.4	1.0	1.4	1.5

The WTP across customized bundles is identical across consumers and the optimal strategy is to offer $p(2/3) = 1.4$. This yields the same outcome as the optimal mixed-bundle solution $[p(a+b) = p(a+c) = p(b+c) = 1.4]$. Note that Result 1(A) does not require identical valuations everywhere, only at points where mixed bundles would be offered. For example, we could introduce another consumer into this example with values $[0.7, 0.7, 0.1]$ without changing the solution, even though this consumer's value of a single favorite good is only 0.7 (versus the 1.0 value of the other three).[10] This example also suggests that the requirement that preferences are identical over all m is sufficient but not necessary.

The conditions in Result 1(B) guarantee that any bundle of a given size that customers choose is already offered in the mixed-bundle solution. The simplest case that satisfies condition (B) is when consumers have similar orderings of goods (i.e., all customers prefer one particular good over the other), although no constraints are needed for their values of the goods. Consider the following example where the preference orderings across goods are the same (assume zero marginal cost):

	a	b	c	$w^i(1/3)$	$w^i(2/3)$	$w^i(3/3)$
1	0.8	0.3	0.1	0.8	1.1	1.2
2	1	0.6	0.3	1	1.6	1.9
3	0.7	0.7	0.7	0.7	1.4	2.1

The optimal solution under mixed bundling in this example is $p(a) = 0.8$, $p(a+b) = 1.4$, and $p(a+b+c) = 2.1$.[11] We can get exactly the same profit with the customized-bundle strategy of $p(1/3) = 0.8$, $p(2/3) = 1.4$, and $p(3/3) = 2.1$. Another interesting observation about this example is that these preferences violate the Spence-Mirrlees single crossing property (SCP)[12] that is commonly assumed for problems of this kind. Thus, SCP is not necessary for customized bundling to replicate the mixed-bundle solution. Note also that it need not be the case for all consumers to agree on their preference orderings over all goods. For instance, consider a two consumer model with valuations $\{0.8, 0.3, 0.1\}$ and $\{1, 0.7, 0.8\}$ over three goods (a, b, c) and zero marginal cost. In this case, the two consumers have different rank order preferences over goods b and c, but the customized-bundle solution with $p(1/3) = 0.8$ and $p(3/3) = 2.3$ still yields the same profit as the mixed-bundle solution with $p(a) = 0.8$ and $p(a+b+c) = 2.3$.

However, it is not hard to construct examples where the conditions in Result 1 fail. Any situation where two same size bundles have different mixed-bundle prices violate both conditions in Result 1. For example, consider a two consumer model with valuations $\{1, 0.5, 0.1\}$ and $\{0.1, 0.4, 1\}$ over three goods (a, b, c) with per-good marginal cost 1/4. Here, the monopolist garners greater profits with two regular bundles with two goods each—goods a and b at a price 1.5, and goods b and c at a price 1.4. There is a profit loss of 0.1 by imposing customized bundling of two goods in this particular example. There are also examples for which the ability to choose any goods in a customized bundle would lead to different consumer choices if prices were maintained. For instance, consider again a two consumer model with preferences $\{0.2, 0.6, 0.6\}$ and $\{0.9, 0.5, 0.6\}$ over goods (a, b, c) with zero marginal cost. Note that these preferences violate both conditions (A) and (B). Here, the mixed-bundle solution is all three goods for 2.0 and goods b

[8] All proofs in this paper are available in an online appendix, available from the authors' websites.

[9] Note that each customer may have a different rank ordering of goods.

[10] However, checking that this is the case is considerably more difficult because it presumes that the mixed bundling solution is already known. The prior example shows common WTP for all values of m and thus is not dependent on knowing the solution to the mixed bundling problem.

[11] Here we assume that a customer will choose the larger bundle when two bundles yield the same surplus.

[12] Single crossing ensures that there is an ordering of consumer valuations. A formal definition of the SCP condition can be found in §3.4.

and c for a price of 1.2 to yield a total profit of 3.2. However, under customized bundling, the monopolist must lower the price of the three-good bundle to be able to serve the second consumer. If they maintained the same prices under customized bundling, the consumer who bought the three-good bundle under mixed bundling would switch to a bundle of only goods a and c to earn 0.3 of additional surplus. The optimal solution becomes three goods for 1.7, and two goods for 1.2 to yield a profit of 2.9.

Interestingly, when consumer valuations for goods are described by a common distribution function, which is a typical assumption in discrete choice and bundling models (see e.g., McFadden 1974, Bakos and Brynjolfsson 1999), the resulting distribution of preferences over goods $W(\mathbf{x})$ yields a common distribution of preferences over customized bundles $w(m)$. Result 2 shows that this relationship holds for quite general distribution functions (essentially all distributions which obey the laws of large numbers).

Result 2. If each of a large number of individual consumer's willingness to pay for a vector of goods $\mathbf{x} \in [0,1]^N$ is given by a vector $\mathbf{v} \in R^N$ ($\mathbf{v} = \langle v_1, \ldots, v_j, \ldots, v_N \rangle$) drawn independently from a common distribution with cumulative distribution function (cdf) $F(\mathbf{v})$ with finite expected absolute value for all goods, there exists an expected WTP function, $w(m)$, for customized bundles that is common across consumers. This function is given by $w(m) = \mathrm{E}[\sum_{k=(1-m)N}^{N} X_{k:N}]$, where $X_{i:N}$ is the ith order statistic from $F(\mathbf{v})$.

Result 2 shows that to calculate consumers' WTP across customized bundles for random distributions, one needs to only calculate $w(m) = \mathrm{E}[\sum_{k=(1-m)N}^{N} X_{k:N}]$. The expression inside the expectation, a linear combination of order statistics, is a special case of a general class of functions called L-estimates (see a survey in Rychlik 1998). This fact will prove useful in later results that we derive for random valuations.

3.4. Solutions

We now conduct a comparative statics analysis of customized bundling. To obtain interesting comparative statics results, we begin by specifying preferences over customized bundles and make an additional assumption about consumer valuations known as the Spence-Mirrlees single crossing property (SCP). This assumption is used in most models of nonlinear pricing and other "hidden type" problems, where comparative statics results are desired. It is important to note that the SCP condition is not required for Results 1 and 2 or for the feasibility of customized bundling. However, without SCP we can make no generalizations about how the optimal bundling solution is affected by changes in consumer values or marginal costs (see Appendix B for an illustration of this problem).[13] If r and s represent bundle sizes (different values of m), and i and j index consumer types, SCP requires that there exists an ordering of consumers such that

$$w^i(r) \geq w^j(r),$$
$$w^i(r) - w^i(s) \geq w^j(r) - w^j(s) \quad \forall\, r > s,\ i > j.$$

Essentially, SCP imposes an ordering of consumer demand over bundles. "Higher type" consumers (higher i in this condition) must place a (weakly) greater value for any given customized bundle than "lower type" consumers and, secondarily, these differences are weakly increasing in bundle size.[14] For all subsequent discussion, assume that customer types are ordered to satisfy this condition.

Let $\{m^{i*}, p^{i*}\}$ denote the optimal offering of the monopolist when there are multiple customer types, and $\{\widehat{m}^i, \widehat{p}^i\}$ represent the (socially optimal) bundle that would be offered to consumer type i if there were no incentive compatibility constraints (that is, if they were the only consumer type being served). Using standard results and proof techniques from the theory of nonlinear pricing (Spence 1980, Armstrong 1996, Rochet and Chone 1998), we can show the following result.

Result 3. A monopolist will offer a set of customized bundles that have the following six properties:

(a) The lowest-type customer that is served is priced at their willingness to pay: $p^{i*} = w^i(m^{i*})$.

(b) The prices for all other bundles are determined to satisfy IC, and leave all consumers except the lowest type with positive surplus (let $i_{\min}$ be the lowest type that is served):

$$p^{i*} = p^{i-1*} + w^i(m^{i*}) - w^i(m^{i-1*}) < w^i(m^{i*}) \quad \forall\, i > i_{\min}.$$

(c) The highest type customer is always served at the size they would have received if they were the only customer segment: $m^{I*} = \widehat{m}^I$.

(d) All other customers receive bundles (weakly) smaller than the bundle size they would have received if they were the only customer segment. These sizes are the greatest values of $m \in [0, 1/N, 2/N, \ldots, 1]$ that satisfy

$$\left(\sum_{j=i}^{I} \alpha^j\right) w^{i\prime}(m^{i*}) - \left(\sum_{j=i+1}^{I} \alpha^j\right) w^{i+1\prime}(m^{i*}) \geq \left(\sum_{j=i}^{I} \alpha^j\right) C'(m^{i*}) \quad \forall i < I.$$

[13] All appendices are available in an online appendix from the authors' websites.

[14] Again, we note that this does not make any additional assumptions about the value of any particular good.

Table 1 Numerical Examples of Result 3

Type	WTP				Optimal bundle size and price for each type			
	$w(m=1/4)$ (or $M=1$ good)	$w(m=2/4)$ (or $M=2$ goods)	$w(m=3/4)$ (or $M=3$ goods)	$w(m=1)$ (or $M=4$ goods)	$MC=0$		$MC=2.5$	
1	2	3	3	3	$\widehat{M}^1=4$ $\hat{p}^1=3$	$M^{1^*}=0$ $p^{1^*}=0$	$\widehat{M}^1=0$ $\hat{p}^1=0$	$M^{1^*}=0$ $p^{1^*}=0$
2	5	7	8	8	$\widehat{M}^2=4$ $\hat{p}^2=8$	$M^{2^*}=1$ $p^{2^*}=5$	$\widehat{M}^2=1$ $\hat{p}^2=5$	$M^{2^*}=0$ $p^{2^*}=0$
3	7	12	14	14	$\widehat{M}^3=4$ $\hat{p}^3=14$	$M^{3^*}=3$ $p^{3^*}=12$	$\widehat{M}^3=2$ $\hat{p}^3=12$	$M^{3^*}=2$ $p^{3^*}=12$
4	9	15	18	20	$\widehat{M}^4=4$ $\hat{p}^4=20$	$M^{4^*}=4$ $p^{4^*}=14$	$\widehat{M}^4=3$ $\hat{p}^4=18$	$M^{4^*}=3$ $p^{4^*}=15$

Notes. $w(m=x/4)$ indicates the WTP for a customer's favorite x product(s) out of four goods. $\{M^{i^*}, p^{i^*}\}$ denotes the optimal offering of the monopolist under customized bundling, and $\{\widehat{M}^i, \hat{p}^i\}$ represents the (socially optimal) bundle that would be offered to consumer type i if they were the only consumer type being served. An optimal offer size with zero (e.g., $\widehat{M}^i$ or $M^{i^*}=0$) means that the customer segment makes no purchase.

(e) There may be, in general, a customer segment such that all customers below that segment are not served (that is, there can be $i_{\min} > 0$ s.t. $m^{i^*}=0$ for $i < i_{\min}$).

(f) The optimal size of the customized bundle is weakly decreasing in marginal cost.

Table 1 illustrates the implications of Result 3 in a setting in which there are four goods, four equally-sized consumer types, and two different assumptions about marginal cost ($MC=0$, $MC=2.5$).

Portions of Result 3 replicate common nonlinear pricing results in our context. First, there is one optimal bundle per type of consumer if that segment is served at all. Second, not all consumers are served with a bundle because, under SCP, it is more profitable to extract additional surplus from the "higher" types than to allow high valuation consumers to select products targeted at lower type segments. In the illustration presented in Table 1, Group 1 is left out of the market ($M^{1^*}=0$) even if the marginal cost is zero, whereas both Group 1 and Group 2 are left out ($M^{1^*}=M^{2^*}=0$) if the marginal cost becomes 2.5. Third, only the highest type consumer is served at their socially optimal bundle size. Other consumer types receive suboptimal bundle sizes designed to discourage high-type consumers from consuming bundles targeted at the lower types. Based on the example above, Group 4 (the highest group) always has $M^{4^*}=\widehat{M}^4$ regardless of marginal costs used in the illustration, while Groups 1–3 receive (weakly) smaller bundles than their unconstrained optimal bundles ($M^{i^*} \leq \widehat{M}^i$ for $i=1,2,3$). Fourth, because the monopolist cannot perfectly price discriminate, all consumers that are served except the lowest type earn some surplus, an information rent due to their hidden type. In the example where marginal cost is zero, only Group 2 (the lowest served) and Group 1 earn zero surplus; all others earn positive surplus because their WTP is higher than the price paid.

Finally, a more subtle observation is that the solution does not yield price linear in bundle size, suggesting that customized bundling may outperform a two-part tariff pricing, a pricing scheme extensively used for some low marginal cost goods such as telecommunications. Intuitively, with a single customer type, two-part tariffs are flexible enough to offer a single point in price-bundle size space. Thus, the two approaches have equal performance. However, this does not hold when there is more than one customer type, as shown in the following corollary:

Corollary 1. *Customized bundling outperforms two-part tariff pricing when there is more than one customer type.*

These results also bring some additional insights that are unique to the customized-bundle problem. First, if cost and WTP are known for each customer segment (whether it is deterministic or the expectation of a random valuation), it is a simple calculation of complexity $O(I)$ to determine the optimal price and bundle sizes that should be offered. This contrasts with the NP-hard mixed-bundle problem of a large number of goods. Second, in this formulation, Result 3(f) shows that the optimal size of the customized bundle is weakly decreasing in marginal cost (note that in Table 1 the bundle sizes decrease as MC changes from 0 to 2.5). This result is interesting because it implies that as the marginal cost per good increases, there is a weakly monotonic shift in the optimal bundling policy from pure bundling to customized bundling to unit sale.

These results provide a general characterization of customized-bundle pricing, problem tractability, bundle sizes, welfare implications, and the relationship between customized bundling and other bundling strategies. In the next two sections, we explore in more detail the relationship between the bundling solution and consumer preferences by making some specific assumptions about cost and consumer valuations.

3.5. Bundling Under a Two-Parameter Preference Function

This section builds on results by Chuang and Sirbu (1999) (thereafter denoted as CS) by considering the case in which different consumers can be described by a WTP function that depends on two parameters: an overall budget constraint or total WTP (b) and the number of goods they value positively (K) (expressed alternatively as the fraction $k = K/N$). Consumers are assumed to have similar utility functions over a rank ordering of goods, given by $w(m) = b \times y(m/k)$, where $y(\cdot)$ captures customers' relative valuations for different goods with $y(0) = 0$, $y(1) = 1$, $y' > 0$, and $y'' \le 0$ over the domain $[0, 1]$.[15] This yields

$$u(m, p_m) = \begin{cases} b \times y(m/k) - p_m & \text{if } m \le k, \\ b - p_m & \text{if } m > k. \end{cases} \tag{3}$$

To gain insights beyond those of Result 3, we focus on a single customer type and consider a specific valuation function:

$$y(t) = (1+a)t - at^2, \quad \text{where} \quad t = m/k \in [0,1] \text{ and } a \in [0,1]. \tag{4}$$

This quadratic form is the simplest function that yields interesting results and provides a local (and possibly global) approximation to arbitrary concave value functions. The parameter a controls the shape of this function and represents our key departure from CS. If $a = 1$, then we have a form equivalent to the CS assumptions (linearly decreasing value in rank order). If $a = 0$, the consumer values all goods equally. We now compare the surplus, profits, and prices for the different bundling schemes.

Pure bundling is a trivial solution as long as the pure bundle is profitable (that is, $b - C(1) > 0$). The monopolist sets price to total value ($p = b$) and extracts all surplus, although when $k < 1$ and $C(k) < C(1)$ it is not efficient because costly goods are bundled that are not valued. The optimal price per good for individual sale (P_{IS}) is found by maximizing profits subject to a constraint that the marginal utility that customers gain by purchasing an additional unit of the good is equated with the prices paid:

$$P_{IS} = \arg\max_{P} \; PmN - C(m) \quad \text{s.t.} \; w'(M) = P, \quad \text{where } m = M/N.$$

Customized bundling has a solution that uses the approach from Result 3 to yield a price for a customized bundle (p_{CB}) of

$$p_{\text{CB}} = \arg\max_{p} \; p - C(m) \quad \text{s.t.} \; w(m) - p \ge 0.$$

The constraint is always binding at optimum, so this problem simplifies to

$$m_{\text{CB}} = \arg\max_{m} w(m) - C(m).$$

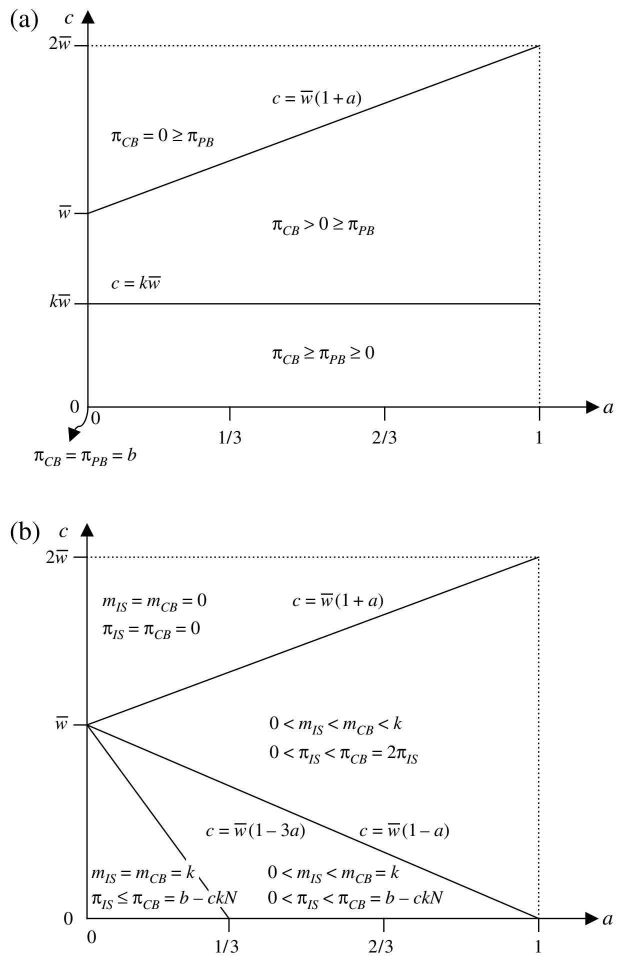
This equation is the same as the maximization of social value, so the customized-bundle solution is efficient. We summarize the solutions and results of the three strategies of individual sale ($m_{\text{IS}}, \pi_{\text{IS}}$), pure bundling ($m_{\text{PB}}, \pi_{\text{PB}}$), and customized bundling ($m_{\text{CB}}, \pi_{\text{CB}}$) in Figures 1(a) and 1(b) (detailed derivations appear in Appendix C). For ease of comparison, we define $\bar{w} \equiv b/kN$ to be the average WTP for the goods that have positive values. Figures 1(a) and 1(b) characterize profit and bundle size for various regions of marginal cost per good (c)[16] and the customer preference parameter over goods (a). Because customized bundling contains both pure bundling and individual sale as extreme cases, it will always weakly dominate. However, the degree of difference depends on marginal cost (c) and the dispersion of values across goods (a). As shown in Figure 1(a), only when marginal cost is zero is customized bundling and pure bundling equivalent in profits. This result continues to hold until marginal costs are equal to $k\bar{w}$. At this point pure bundling is no longer feasible but customized bundling is still profitable. Finally, at $\bar{w}(1+a)$ customized bundling is no longer feasible. Altogether, these results suggest that the profitable region (over marginal cost) of customized bundling expands as a increases (i.e., when there is increasing difference in valuations of goods).

In Figure 1(b), individual selling and customized bundling are equivalent in number of goods sold, and both are efficient if marginal cost is very low and dispersion of valuation across goods is low [$C' = c < \bar{w}(1-3a)$]. Both also achieve the same profit level when $a = 0$. As a departs from zero but is smaller than $\bar{w}(1-3a)$, individual selling is efficient but not

[15] One can think of $y(t)$ as the proportion or fraction of total budget that a customer is willing to spend on the top t percent of the goods she positively values. Intuitively, y is an increasing and concave function of t. Consumers can have different rank ordering of the goods and we do not specify any assumptions on the valuation of each particular good.

[16] We assume that $C(m) = cmN$.

Figure 1 (a) Pure Bundling vs. Customized Bundling for Different Marginal Cost and Customer Preference Parameter, (b) Individual Selling vs. Customized Bundling for Different Marginal Cost and Customer Preference Parameter



profit maximizing because the provider is leaving consumers with significant consumer surplus.[17] An interesting observation is that the market demand can be fully served with customized bundling for a values three times as large as is the case of individual selling for the same level of marginal cost (in other words, customized bundling can accommodate greater degrees of customer heterogeneity). Finally, as marginal costs increase, the size of the customized bundle decreases until marginal cost is so high that bundling is infeasible. On the other hand, for intermediate values of marginal cost, $\bar{w}(1-a) < c < \bar{w}(1+a)$, customized bundling is two times more profitable than unit sale under these assumptions.

Overall, these results collectively suggest that: (1) customized bundling becomes favorable to alternative bundling approaches as marginal cost increases and consumer valuations over different goods become more heterogeneous; and (2) customized bundling is feasible (in the sense of being profitable for a monopolist) over a larger region of the parameter space than pure bundling and individual sale.

These results are derived for a single group of customers. The contrast will only increase if we allow multiple customer segments because customized bundling can offer tailored bundles to each segment. Such a strategy is not possible with individual sale or pure bundling without some additional segmentation mechanism.

3.6. Customized Bundling Under Random Valuation

Earlier, we showed that simple customized-bundle solutions can arise when valuations are drawn from a common distribution (see Result 2). Given the importance of these types of assumptions in the bundling literature (see, e.g., BB), we explore these types of models in greater detail in the customized-bundle context. Our results will be derived for the case of a single-type common distribution for all consumers, although the results can be extended to the case of multiple types using Result 3 when the resulting expected valuation functions satisfy SCP. For this analysis we will fix N, the number of goods in the population (BB consider results where N can vary), because this makes pure bundling a special case of customized bundling ($M = N$). We retain the assumptions of BB of identically distributed v_i (the value of the ith good) with finite expected absolute value and constant marginal cost per good (which may be zero). For the following results, it is useful to define the "quantile" or "inverse distribution function" of a distribution function $F(t)$ as $Q_F(z) = \sup(t\colon F(t) \le z)$.

This setup enables us to bound the value of customized bundles for arbitrary distribution functions (including dependent valuations):

Result 4. If the valuation for any individual good is drawn from an identical but possibly dependent distribution $F(\cdot)$ with finite mean (μ), then $\int_{1-m}^{1} Q_F(z)\,dz \ge w(m) \ge mN\mu$.

There are two interesting insights from Result 4. First, the upper bound can sometimes serve as a reasonable approximation for the value of customized bundles, because it is approximately the average value of goods above the mth percentile in value. Simulation results suggest that this approximation is good for common distributions (uniform, normal, logistic, and exponential), especially when valuation of different goods is not too dependent. Second, customized-bundle value always (weakly) exceeds the mean value of the same number of goods, suggesting that per-good values of customized bundles will often dominate per-good values of pure bundles.

[17] Note, however, that one can implement a two-part tariff with an entry fee to extract this surplus and get the same profit as can be achieved by the customized bundling approach.

The strict lower bound holds when $m = 1$ or valuations of goods are perfectly correlated.

With additional assumptions on the distribution, we can apply the theory of L-estimates to obtain a number of additional general results. For example, we obtain exact results if we further assume independence of goods valuations, that is, $F(\mathbf{v}) = \prod_{i=1}^{N} F(v_i)$. This assumption yields an explicit expression for $w(m)$ in terms of the distribution quantile function.

Result 5. If the valuation of individual goods is independently and identically distributed (i.i.d.) with quantile function $Q_F(z)$, then $w(m) = \int_0^1 Q_F(z) \cdot \sum_{i=mN}^{N} N_{i:N}(z)\,dz$, where

$$N_{i:N}(z) = N\binom{N-1}{i-1} z^{i-1}(1-z)^{N-i}$$

(the Bernstein polynomials).

This expression can be used to numerically calculate the values of the consumer's WTP for arbitrary distribution functions and is solvable in closed form for some distributions such as the uniform and the exponential. Moreover, it can be used to generate comparative statics results for distributions in the location-scale family, which includes most of the common distributions assumed in prior work such as the exponential, normal, and uniform. Location-scale distributions are distributions where the quantile function can be described with two parameters (a, b) with $Q_F(z; a, b) = a + bQ_F(z; 0, 1)$. The parameter a is referred to as the location (proportional to the mean) and b as the scale (proportional to the variance).[18] For i.i.d. distributions we can now derive the relationship between the optimal customized bundle size (m^*) and the location and scale parameters.

Result 6. Let the valuation for any individual good be drawn independently from a distribution $F(x)$ with mean (μ), in the location-scale family with location a and scale b. Then,

(a) m^* increases weakly in a.

(b) For general distributions m^* increases weakly in b if $\mathrm{E}[X_{M+1:N}] < c$, where M is the lowest-order statistic of the standard distribution for $F(x)$ ($a = 0$, $b = 1$) with nonnegative expected value. m^* decreases weakly in b if $\mathrm{E}[X_{M-1:N}] > c$.

(c) Profits are weakly increasing in m^*.

For any fixed marginal cost, an increase in the location parameter (a) simply shifts the valuation curve outward in marginal value-size space, increasing optimal bundle size (unless the optimal bundle is already the pure bundle). The intuition behind the scale parameter result is somewhat more complex. As scale (b) increases, the distribution becomes more dispersed—higher-order statistics become larger and lower-order statistics become smaller. If the optimum lies in a region where the order statistics are increasing in variance (i.e., when $c > \mathrm{E}[X_{M+1:N}]$, or in other words, when the optimal bundle only includes the very highest-valued goods), then increasing scale raises the optimal size of the bundle. If the optimum lies in a region where the order statistics are decreasing in scale, the optimal bundle size is decreasing in scale. The conditions in Result 6(b) guarantee that the optimal point does not "change sides" as the scale parameter varies.

This result shows an interesting relationship between pure bundling and customized bundling. When it is feasible to have a pure bundling solution (the average value greater than marginal cost), then greater variance will decrease the performance of pure bundling relative to customized bundling because it means that the lowest-valued goods in the bundle become even less valued with increasing variance. This result augments the explanation of BB that greater variance slows convergence of consumer valuations to the mean, leaving consumers with more surplus. In addition, when marginal cost is high enough that pure bundling is infeasible ($\mu < c < \mathrm{E}[X_{N:N}]$), increasing variance actually leads to larger customized bundles (which are, however, always smaller than the pure bundle) and greater bundling profits in contrast to the results of BB.

We can relax the independence assumption if we restrict the distribution of value to be multivariate normal with common correlation (ρ), for which closed-form expressions for the order statistics are available. These results are given in Result 7, using the same notation introduced in Result 6(b).

Result 7. If good valuations are described by a multivariate normal distribution with common correlation, the optimal bundle size and total bundle profits decrease with correlation among goods if $\mathrm{E}[X_{M+1:N}] < c$, and increase with correlation if $\mathrm{E}[X_{M-1:N}] > c$, where M is the median.

This result indicates that negative correlation acts similarly to variance, with negative correlations raising the value of the highest-valued goods but also decreasing the value of the lower-valued goods. On the other hand, a large positive correlation results in less dispersed order statistics; under perfect correlation there is no variance in the observed order statistics. This yields another contrast with the results in BB—in the region where pure bundling is feasible—the efficiency gains resulting from convergence to the mean from negative correlations are offset by the marginal goods being lower in

[18] Note that the a parameter here is not the same as the preference shape parameter in §3.5. We retain this notation for consistency with prior research.

value, thereby favoring customized bundles over pure bundles.

These types of arguments also extend to situations in which goods can be complements or substitutes. Following previous models of complementary goods in bundling (Bakos and Brynjolfsson 2000, Venkatesh and Kamakura 2003), we represent complementarity or substitutability by allowing mean valuation to depend on bundle size $EW(\mathbf{x}) = M^{\alpha}\mu$, where $M = \mathbf{x} \bullet \mathbf{1}$ is the number of goods in the bundle. The parameter α encodes shape, with $\alpha < 0$ indicating that the goods are substitutes and $\alpha > 0$ indicating complementary goods. Our prior results on independent valuations would correspond to $\alpha = 0$.

Clearly, a given bundle is most valuable when it contains complements in this formulation. This observation suggests that the parameter α acts just like the location parameter considered in Result 6. As α increases, overall valuation for a given bundle size increases, which increases the optimal bundle size ceteris paribus. This result, summarized in Result 8, is consistent with prior results by BB and others that complementarities create additional incentives for bundling.

Result 8. The optimal bundle size and total bundle profits increase when goods are complements.

4. Summary and Conclusion

We have analyzed an alternative bundling mechanism for low marginal cost goods that allows a consumer to choose up to M of their preferred goods from a larger set N for a fixed price p. In some circumstances, including those used in prior bundling work, the full bundling problem can be reduced to a customized-bundle problem. This greatly simplifies the complexity of the problem, especially for large numbers of goods, and enables known results on nonlinear pricing to be applied to otherwise intractable bundling problems. In addition, because CB nests individual sale and pure bundling as special cases, we can compare the relative performance of these different pricing approaches to CB.

We show that CB is particularly attractive when consumers are budget constrained, marginal costs are low but nonzero, and consumers' valuation is concentrated on a relatively small number of goods (though not necessarily the same ones across the consumer population). In addition, for the case in which consumer valuations are generated by identical distributions, we also show that uncertainty about consumers' valuations (variance) makes customized smaller bundles more attractive when marginal costs are low. Moreover, the optimal customized bundle size increases in variance when marginal costs are relatively high (in contrast to BB). Similar results also hold when goods are negatively correlated. We also replicate prior results that complementarity among goods provides greater incentives for bundling.

Customized bundling can be especially advantageous for monopolists who are selling large numbers of high-value goods to consumers with heterogeneous preferences. Examples might include motion pictures, high-quality digital music, or modular packaged software to small enterprise customers[19] (e.g., enterprise resource planning suites such as SAP's R/3 system). Pure bundling will likely prevail if marginal costs are negligible and consumer heterogeneity is limited. Individual sale is favourable under conditions of high marginal costs. However, even in these cases, customized bundling may be attractive because it enables price discrimination through bundle size, a capability not possible in a pure bundling approach when third-degree price discrimination cannot be enforced.

As a relatively novel approach to bundling, there has been limited understanding of the benefits and design heuristics of this approach. This may explain why CB is not as extensively used as our results might predict. Nonetheless, there is evidence that this approach has proved advantageous in practice. In a field experiment, MacKie-Mason et al. (2000) found that librarians shifted toward purchasing journals through customized bundling (or "generalized subscriptions" in their terminology) when this option was offered along with other more traditional pricing schemes. Their consumption of customized bundles increased over time relative to other pricing approaches, even when it was likely that preferences over journal articles were largely unchanged.

We have also identified a number of other examples used in a nonresearch context. For instance, firms offering modular engineering software often license on the basis of number of modules used, an approach which is essentially customized bundling. There are also the well-known "10 CDs for a $1" promotions by firms such as Columbia House, which represent the purchase of a customized bundle of around 14 CDs for approximately $75 (once contractual requirements are met). At least one online movie rental club (netflix.com) currently uses a customized-bundle scheme—Netflix's pricing scheme allows users to choose different plans that enable them to simultaneously borrow N videos for $p(N)$ dollars per month where multiple values of N are allowed (currently 2, 3, 4, 5, and 8). The *New York Times* has experimented with a bundle-pricing scheme for access to its article archives (four pricing options ranging from

[19] Large-scale enterprise software licenses are often negotiated individually so a marketwide pricing schedule has less importance. Smaller customers are likely to receive license terms closer to standard packaged software pricing.

25 articles for \$25.95 to a single article for \$2.95). Pressplay.com licenses music for download using a similar scheme. O'Reilly and Associates, a publisher of technical books, licenses their digital content by selling packages ("tokens") that include a certain number of downloads. While most of these firms are still in the phase of exploring or experimenting with customized bundling, our analysis suggests that customized bundling is beneficial. Our results further offer some guidelines for firms to evaluate their present pricing scheme.

As nascent "direct to consumer" sale of digital goods becomes more common, we expect that customized bundling will become more prevalent. Indeed, a number of prominent online businesses currently distribute digital content on a unit sale basis or other more traditional pricing schemes—but our results suggest that at least from a pure pricing standpoint, they could earn greater profits by adopting a customized-bundle approach with the right design. In addition, there is also great potential for customized bundling to be used for new digital goods or services, such as search results, reviews or ratings, automatic agent services, consultations, textbook chapters, and product listings, to name a few. This paper has offered some guidance for firms to design appropriate pricing strategy.

Moreover, customized bundling need not be limited to information goods, because some physical products or services share the essential properties of information goods such as low marginal costs. For example, pizza delivery restaurants offer three-topping pizzas for a fixed price, and some fast-food restaurants have a "bundled" side dish selection—consumers pick two or three side dishes from a specified set. The Pittsburgh Symphony also sells a customized bundle of tickets called "Flex-8," whereby a customer can attend any eight concerts throughout the year. Airlines for many years have sold tickets that enable customers to choose flights up to a total mileage limit. Collectively, these examples suggest a wide range of applicability of customized bundling.

In addition to the potential for practical use, our customized-bundle analysis provides another simplification to the general problem of optimizing bundle compositions that may be appropriate in some circumstances. Given the complexity of the general mixed-bundle problem, there has been tremendous interest in the marketing, management, computer science, and economics communities for approaches that yield tractable analytic bundling solutions.

An online appendix to this paper is available at http://mansci.pubs.informs.org/ecompanion.html.

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